Probability of ruin in continuous time in a finite horizon with variable premiums

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Introduction

This is model to estimate the probability of ruin in continuous and finite time for a compound Poisson risk process where the premium rate is constant throughout each year but can change at the start of each year. This method was developed by [Afonso et al., 2009] for large portfolios. General references are text by [Seal, 1969], [Bühlmann, 1970], [Gerber, 1979], [Bowers et al., 1997] or more recently [Asmussen, 2000].

Comments, suggestions or bug reports ?

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Goal

Evaluate (numerically) the ruin probability in continuous and finite time for a Compound Poisson risk process

Assumptions:

- The occurrence of claims is a Poisson Process.
- The premium is received at a constant rate during each year, but may change every year.

Definitions

We consider a risk process over an n-year period. We denote by $S(t)$ the aggregate claims up to time t, so that $S(0) = 0$, and by Y_i the aggregate claims in year i, $i = 1, \ldots, n$, so that $Y_i = S(i) - S(i-1)$. We assume that $\{Y_i\}_{i=1}^{n}$ is a sequence of *i.i.d.* random variables, each with a compound Poisson distribution whose first three moments exist. We denote by λ the Poisson parameter for the expected number of claims each year and by $f(s)$ the probability density function (*pdf*) of $S(s)$ for $0 < s \leq 1$.

Definitions

Let P_i denote the premium charged in year i and let $U(t)$ denote the insurer's surplus at time t, $0 \leq t \leq n$. We assume premiums are received continuously at a constant rate throughout each year. The initial surplus, u $(= U(0))$, and the initial premium, P_1, are known. For any time t, $0 \leq t \leq n$, $U(t)$ is calculated as follows:

$$U(t) = u + \sum_{j=1}^{i-1} P_j + (t - i + 1)P_i - S(t)$$

where i is the integer such that $t \in [i-1, i[$, and where $\sum_{j=1}^{0} P_j = 0$.

Definitions

For $i \geq 2$, the premium P_i and surplus level $U(i)$ are random variables since they both depend on the claims experience in previous years. Where we wish to refer to a particular realization of these variables, we will use the lower case letters p_i and $u(i)$, respectively. The probability of ruin in continuous time within n years is denoted by $\psi(u, n)$ and defined as follows:

$$\psi(u, n) \stackrel{def}{=} \Pr(U(t) < 0 \text{ for some } t \in]0, n])$$

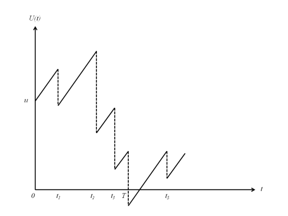

Figure: Sample path of surplus

Inputs

Figure 2 shows the inputs needed to the simulation procedure. The **Initial surplus** is the available amount of surplus for the portfolio. **Expected premiums received during the next 4 years** are the expected amounts to be received in the next years. The method also needs the first moment of the Poisson process. **Random seed from 1 to 1000** allows you to choose a different seed for the simulation procedure in order to obtain the same result on the next simulation.

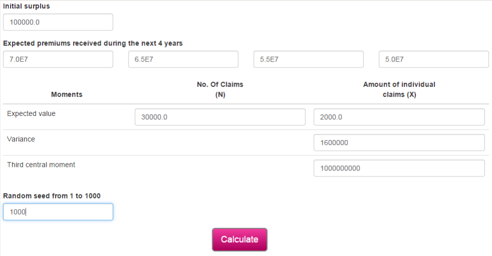

Figure: Inputs

Outputs

Figure 3 shows the path of the premiums, the expected path of claims and surplus. The bar indicates the evolution of the 1.000 runs of the simulation procedure.

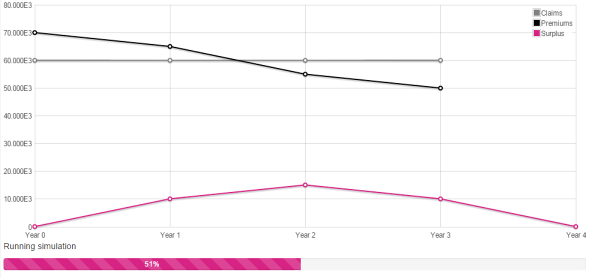

Figure: Expected path of the surplus

Outputs

Figure 4 shows the results of the simulation. It indicates the average and the variance of the 1.000 runs that estimate $\psi(u, 2)$ and $\psi(u, 4)$. In this case as the expected path for the surplus decreases from year 2 forward $\psi(u, 2) < \psi(u, 4)$. $\psi(100.000, 2) = 0,0785$ means that the probability of ruin in two years time with an initial surplus of 100.000 in the portfolio described in Figure 2 is 7,85%. $\psi(100.000, 4) = 51,7\%$. One should increase the surplus during the year 3 or the premiums in order to decrease the probability of ruin.

	Average	Variance
Ruin probability during 2 years horizon	7,84669e-2	4,37187e-7
Ruin probability during 4 years horizon	5,16765e-1	2,14597e-4

New simulation

Figure: Results of the simulation

Other Results

- If the claim number is not a Poisson Process, for example in Automobile portfolio in the presence of a Bonus Malus System (BMS), this method does not apply. The model has to be adjusted.
- Compare the impact on the ruin probabilities of different BMS.
- Evaluate the ruin probabilities of a Automobile portfolio in the classical model (closed portfolio) and using stochastic vortices (open portfolio).
- Compute any within the year ruin probability and the ultimate ruin probability for the portfolio.
- Change the claim amount distribution during the time horizon.

Please contact us to evaluate your personal case at geral@magentakoncept.com

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